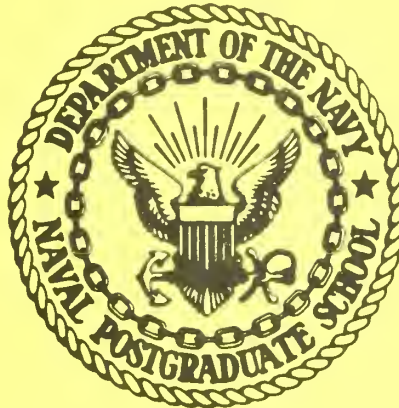


NPS55-78-013

# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



AN ANALYSIS OF BOLTER-HOLE SPACING IN  
AIRCRAFT CARRIER LANDINGS

by

Kneale T. Marshall

May 1978

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NPS55-78-013	2. GOVT ACCESSION NO. AD-A068585	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) An Analysis of Bolter-Hole Spacing in Aircraft Carrier Landings		5. TYPE OF REPORT & PERIOD COVERED Technical
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Kneale T. Marshall		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, Ca. 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, Ca. 93940		12. REPORT DATE May 1978
		13. NUMBER OF PAGES 51
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Aircraft Landing Strategies Carriers		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)  This paper presents an analysis of aircraft landing strategies on aircraft carriers. Optimal bolter-hole spacings are determined for various measures of effectiveness, and a short discussion is included on the use of automatic landing systems.		



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## O. INTRODUCTION

The purpose of this paper is to present and analyze a model of the use of various strategies in landing patterns when trying to recover a squadron of aircraft returning to an aircraft carrier. Section 1 contains a brief description of the landing procedure and why a problem arises. Section 2 describes the basic model from which we derived basic equations of interest. In Sections 3, 4 and 5 we develop expressions for three different measures of effectiveness, the expected time to recover all aircraft, the variance of this time, and the total expected fuel used in recovery, respectively. The expressions each require the determination of certain probabilities before they can be used for numerical calculations. Expressions for calculating these probabilities are given in Section 6. Finally, in Section 7, we present numerical calculations to demonstrate how the model can be used to investigate various landing strategies.

## 1. CARRIER LANDING PROCEDURES

Figure 1 is an illustration of how carrier landings are performed. The aircraft (usually about 30) are marshalled in a holding pattern 20 nautical miles behind the carrier, separated by 1000 feet in elevation, starting at 5,000 feet. Each aircraft is given a precise time and position at which to leave the holding pattern and follow a glide-path to the landing deck of the carrier. These times and positions are such that nominally the aircraft reach the landing point at intervals of 1 min 15 secs. Thus with 30 aircraft, if we count time from when the first aircraft reaches the landing point, and if all aircraft land successfully on their first attempt, it would take 36 min 15 secs to recover all the aircraft.

A problem arises because with carrier landings not every attempt is successful. There is some probability that an aircraft is either waved off or misses the arresting cable. We say that an aircraft which does not land successfully on a given attempt bolters. Aircraft which bolter join a queue at 1,200 feet to await a further landing attempt. This queue we call the bolter queue. An aircraft in the bolter queue cannot make another landing approach until there is room for it in the landing pattern. If all aircraft are scheduled so that they arrive at the landing point every 1 min 15 secs, then aircraft in the bolter queue must wait until all aircraft have made their initial attempt before they can make their second or subsequent attempt.



Holding Pattern ( To 34000' )  
( 30 Aircraft )

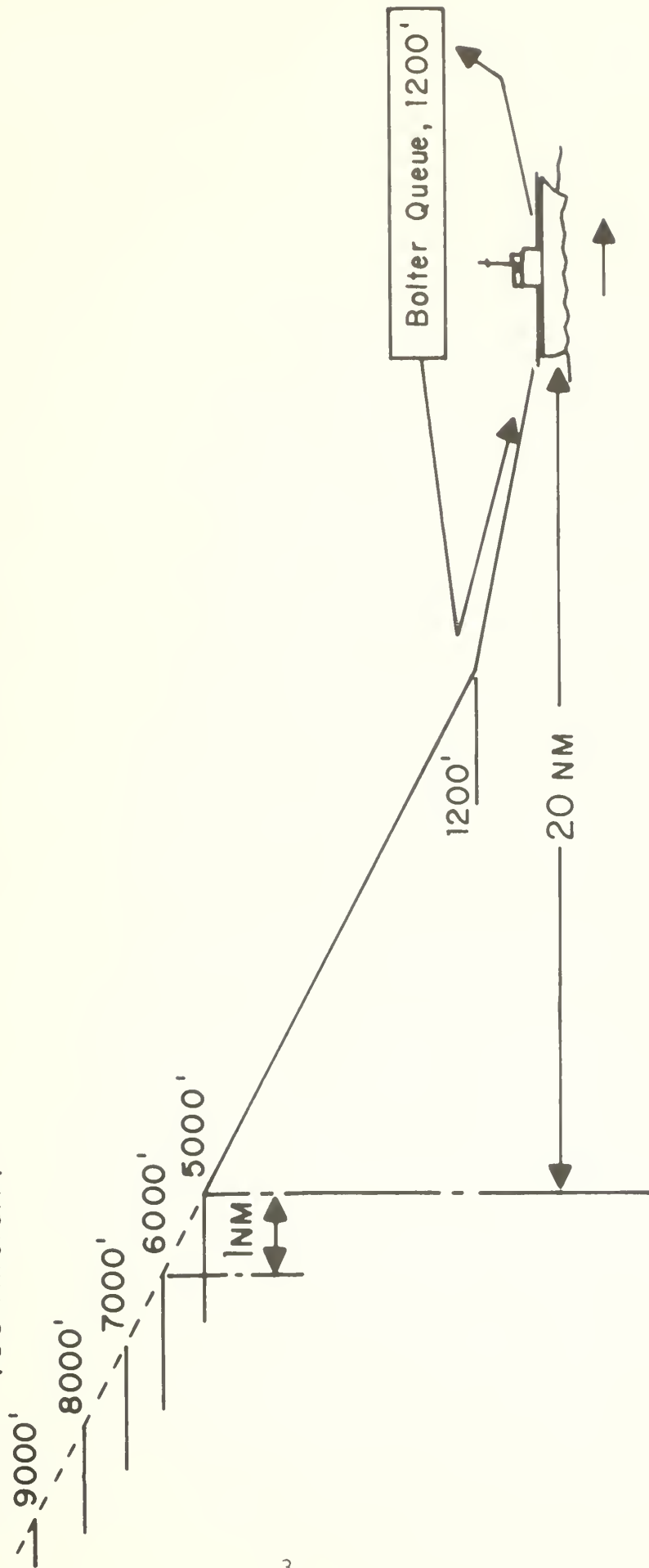


Figure 1: Aircraft Holding Pattern, Approach, and Bolter Queue.

To prevent undue delays in the bolter queue it is common practice to leave "bolter holes" in the landing pattern. A hole is simply a 1 min 15 secs time in which there is no aircraft scheduled to land from the holding pattern. Figure 2 illustrates a landing strategy with eight aircraft, with two holes left in the fourth and eighth positions. These positions could be filled by aircraft from the bolter queue if necessary. If the bolter queue is empty when a hole appears, then the hole remains unfilled.

The probability that a landing attempt is successful depends on many factors, such as time of day (day or night landing), weather, moon state if landing at night, sea state, and experience of the squadron at sea. For a seasoned squadron which has been at sea for some time, landing with calm seas on a clear afternoon, the probability of a successful landing will be high. But with a squadron only recently at sea landing in rough weather at night with no moon or in rain, the probability can be quite low.

Before formulating the model a few further points of clarification should be made. Readers may think (as the author originally did) that the problem is one of sequential decision making. If an aircraft bolters, then one should update all the landing times of the aircraft still in the holding pattern in order to reschedule the boltered aircraft. This may be an interesting model, but it cannot be implemented in practice for many reasons, including safety and an unacceptable increase in radio traffic. Thus the problem is to decide a priori what sequence of scheduled landings and holes to use for a given

A – SCHEDULED AIRCRAFT FROM HOLDING PATTERN  
H – BOLTER HOLE

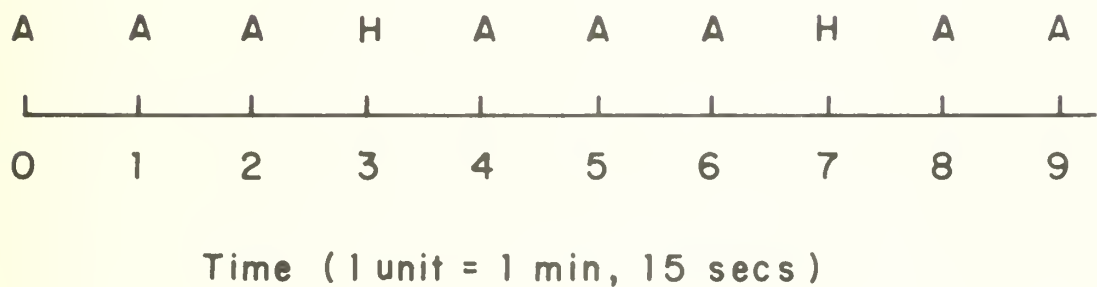


Figure 2: Example of a Landing Strategy with  
8 Aircraft.

---

landing probability and values of other model parameters. The author has written a related paper (Marshall [2]) which deals with the model building problems as opposed to the technical ones of the final model used which are described below.

Those familiar with the real landing problem will recognize some simplifying assumptions which have been made. It is assumed in this paper that all aircraft eventually land; none are sent to shore or to aerial tankers for refuelling. This point is discussed in Section 7. Also it is assumed that if a hole is present and the bolter queue is not empty, it is always possible to place one of the boltered aircraft in the hole.

$$n(t) = n(t-1) + q_1 \quad (1)$$

and

$$n^{(2)}(t) = n^{(2)}(t-1) + 2q_1 n(t-1) + q_1, \quad (2)$$

with

$$n(0) = n^{(2)}(0) = q_1.$$

Now let  $t$  be the time of a bolter hole. We assume that if the bolter queue is not empty at  $t-1$ , an attempt is made to land an aircraft from this queue in the hole at  $t$ . If it does not land it returns to the bolter queue. Thus

$$N(t) = N(t-1) - U(t),$$

where

$$\begin{aligned} U(t) &= 1 \quad \text{if both } N(t-1) > 0 \quad \text{and the aircraft lands,} \\ &= 0 \quad \text{otherwise.} \end{aligned}$$

Note that for this case  $t$  cannot be zero. Since

$$P[U(t) = 1] = p_2 P[N(t-1) > 0],$$

we have

$$n(t) = n(t-1) - p_2 P[N(t-1) > 0], \quad (3)$$

$$n^{(2)}(t) = n^{(2)}(t-1) - 2p_2 n(t-1) + p_2 P[N(t-1) > 0], \quad (4)$$

for the case where  $t$  is a bolter hole.

Equations (1)-(4) are used in the next three sections to determine expressions for the three measures of effectiveness described in Section 1.

### 3. EXPECTED RECOVERY TIME ( $r$ )

Let  $R$  be the time from  $t = 0$  to when the last aircraft is successfully recovered. For a given strategy  $S$  with  $K$  elements and with  $m$  aircraft to be recovered, the smallest value that  $R$  can be is  $m + K - 1$ . Let  $B$  be the time from  $t = m + K - 1$  until the bolter queue is empty. Then clearly

$$R = m + K - 1 + B , \quad (5)$$

and on taking expected values, if we let  $r = E[R]$  we obtain

$$r = m + K - 1 + E[B] . \quad (6)$$

Since it will take a geometric number of attempts to recover each aircraft from the bolter queue after  $t = m + K - 1$  with success probability  $p_2$ , we have

$$E[B] = \frac{n(m + K - 1)}{p_2} . \quad (7)$$

For a given strategy  $S = \{s_1, s_2, \dots, s_K\}$  we now use equations (1) and (3) to solve for  $n(m + K - 1)$ . First define

$$\alpha_i = P[N(s_i - 1) > 0] , \quad i = 1, 2, \dots, K . \quad (8)$$

Thus  $\alpha_i$  is the probability that the bolter queue is non-empty when the  $i$ -th hole appears. The structure of  $\alpha_i$  is described later, in section 6, but notice that it depends on  $s_1, s_2, \dots, s_i$  and on  $p_1$  (and  $p_2$  when  $i > 1$ ). Using (8), (1) and (3) it is straight forward but tedious to show that

$$n(m + K - 1) = mq_1 - p_2 \sum_{i=1}^K \alpha_i . \quad (9)$$

Writing  $r$  as  $r(S)$  to indicate its dependence on the strategy  $S$ , and using (9), (7) and (6) we find that

$$r(S) = \frac{m(p_2 + q_1)}{p_2} - 1 + \sum_{i=1}^K (1 - \alpha_i) . \quad (10)$$

In order to use (10) for calculating  $r$ , we need to be able to determine the  $\alpha_i$ ,  $i = 1, \dots, K$ . Expressions for these are derived below in Section 6. But we do not need to calculate them to determine the strategy which minimizes  $r$ . Since  $\alpha_i$  is a probability, all terms in the sum on the right-hand side of (10) are non-negative, and this sum is the only part of the expression which is a function of the strategy used. Thus to minimize  $r$  we set  $S = \emptyset$ . That is, leave no bolter holes in the landing pattern.

Equation (10) has an interpretation which makes obvious this result. The first two terms in (10) give the total expected number of attempts it will take to land  $m$  aircraft (the  $-1$



comes from our choice of starting at  $t = 0$  with a scheduled landing). The summation term gives the expected number of empty holes which will occur using strategy  $S$ . Since there is always a positive probability (often very small in practice as we shall see) that a bolter hole will not be filled, it is optimum not to leave any.

#### 4. VARIANCE OF THE RECOVERY TIME ( $v$ )

An important factor in any process is often its variability. Uncertainty as to the time to recover all aircraft causes problems in the scheduling of other functions aboard the carrier. Thus it is important that we know how the variance of the recovery time changes with the strategy  $S$ . It may be possible to choose a strategy which gives significant reduction in recovery time variance without increasing the expected recovery time substantially.

From equation (5), if we define  $v$  to be the variance of  $R$ , then

$$v = \text{Var}[B] . \quad (11)$$

Thus all the variance in the recovery time  $R$  is in the recovery from the bolter queue after the scheduled landing pattern has been completed.

A simple way to proceed in the evaluation of  $v$  in (11) is to use the conditional variance relation

$$\text{Var}[B] = E[\text{Var}[B|N(m + K - 1)]] + \text{Var}[E(B|N(m + K - 1))] .$$

Observing again that each aircraft in the bolter queue at the end of the schedule will take a geometric number of attempts to land, then

$$\text{Var}[B] = (n(m + K - 1)q_2 + \text{Var}[N(m + K - 1)])/p_2^2. \quad (12)$$

An expression for  $n(m + K - 1)$  is given in (9), but we need to determine one for  $\text{Var}[N(m + K - 1)]$ , or equivalently, for  $n^{(2)}(m + K - 1)$ .

Recall that equations (2) and (4) in Section 2 give recursive expressions for  $n^{(2)}(t)$  when there is a scheduled landing or a bolter hole at  $t$  respectively. Straight forward but tedious algebra using (2) and (4) with a given strategy leads to the expression

$$n^{(2)}(m+K-1) = mq_1 + 2q_1 \sum_{t=0}^{m+K-2} n(t) - 2(q_1 + p_2) \sum_{i=1}^K n(s_i - 1) + p_2 \sum_{i=1}^K \alpha_i. \quad (13)$$

By using (1) and (3) we obtain

$$\sum_{t=0}^{m+K-2} n(t) = \frac{(m-1)mq_1}{2} + q_1 \sum_{i=1}^K (s_i + 1 - i) - p_2 \sum_{i=1}^K (m + K - 1 - s_i) \alpha_i. \quad (14)$$

and

$$\sum_{i=1}^K n(s_i - 1) = q_1 \sum_{i=1}^K (s_i + 1 - i) - p_2 \sum_{i=1}^K (K - i) \alpha_i. \quad (15)$$

From (15), (14), (13), (12) and (11), after some algebra one finds

$$\begin{aligned}
v(S) = & \frac{mq_1(1+q_2-q_1)}{p_2^2} - \frac{2q_1}{p_1} \sum_{i=1}^K (s_i + 1 - i)(1-\alpha_i) \\
& + 2 \sum_{i=1}^K (K-i+\frac{1}{2})\alpha_i - \left( \sum_{i=1}^K \alpha_i \right)^2, \quad (16)
\end{aligned}$$

where we have written  $v(S)$  to remind the reader of  $v$ 's dependence on the strategy  $S$ . Equation (16) can be used to calculate the variance of  $R$  once the probabilities  $\alpha_i$ ,  $i = 1, 2, \dots, K$  have been determined for the given strategy.

It is not clear from (16) how the variance changes with the strategy. We note that  $v(\emptyset)$  is given by the first term which is simple to evaluate. Note also that if we replace  $\alpha_i$  by 1.0 for every  $i$ , the second term drops out and the last two terms are each equal to  $K^2$  and cancel. Thus for strategies where it is almost certain there will be an aircraft in the bolter queue when a hole occurs, there is virtually no reduction in the variance over the no-hole strategy.

One can argue that a strategy can be found that makes  $v$  arbitrarily small (at the expense of making  $r$  arbitrarily large). For example, suppose we use the (completely unrealistic) strategy of leaving 20 holes after each scheduled landing. With a realistic value of  $p_1$  the probability of having no aircraft in the bolter queue at the end of the schedule would be extremely close to 1.0. Thus we would have virtually no

variance in  $B$ , and hence in  $R$ . The point of this example is to show that searching for a strategy which minimizes variance would be futile. However one can use (16) together with (10) to investigate tradeoffs between  $r$  and  $v$  for various strategies. This is illustrated computationally in Section 8.

## 5. EXPECTED FUEL CONSUMED IN RECOVERY (f).

An important factor in determining which landing strategy to use is that aircraft use fuel at a considerably higher rate in the bolter queue at 1200 feet than in the holding pattern at elevations greater than 5000 feet. In this paper we assume that  $\phi$  is the ratio of the fuel consumption per unit time in the bolter queue to that in the holding pattern. This ratio varies somewhat with aircraft type but we assume that it can be estimated for a given application. Typical values for modern Navy aircraft are in the range 1.3-1.5. Rather than work with units of actual fuel consumption (lbs/minute, and total lbs used in the recovery), we work in units of fuel consumed in one time unit in the holding pattern.

From equation (5) we see that there are two distinct parts to the total recovery time, the scheduled pattern of fixed length  $m + K - 1$ , and the bolter queue recovery time following this of random length  $B$ . We calculate the total expected fuel consumed by considering these parts separately. Furthermore, we break down the fuel used in the fixed interval  $(0, m + K - 1)$  into that used in the holding pattern and that used in the bolter queue.

### Total Fuel Used in the Holding Pattern

Since  $t = 0$  is the time that the first aircraft reaches the landing point, this aircraft uses no fuel in the holding

pattern during the measured time interval. The second uses 1 unit if  $t = 1$  is not a bolter hole. In fact for  $0 \leq i < s_1$ , aircraft  $(i + 1)$  uses  $i$  units. In general, the aircraft which is scheduled to land at time  $t$  uses  $t$  units in the holding pattern. If  $h$  is the total fuel used in the holding pattern, then for strategy  $S$  it is easily shown that

$$h(S) = \frac{(m + K - 1)(m + K)}{2} - \sum_{i=1}^K s_i .$$

For the no-hole strategy we have  $h(\emptyset) = [m(m-1)]/2$ . Using this we can rewrite  $h$  in the form

$$h(S) = \frac{m(m-1)}{2} + \frac{K(2m + K - 1)}{2} - \sum_{i=1}^K s_i . \quad (17)$$

This expression is independent of  $p_1$  because once a strategy is fixed, the actual realization of successful landings has no effect on  $h$ .

#### Expected Fuel Consumed in the Bolter Queue During the Landing Schedule

Because of our choice of units, the total expected fuel consumed in the bolter queue during the landing schedule of length  $(m + K - 1)$  is equal to  $\phi$  times the total expected number of aircraft-periods spent in the bolter queue during this period. Let  $a$  denote this total expected fuel. Then

$$a(S) = \phi \sum_{t=0}^{m+K-2} n(t) ,$$

and by using equation (14) we obtain

$$a(S) = \frac{(m-1)m q_1 \phi}{2} + q_1 \phi \sum_{i=1}^K (s_i + 1 - i) - p_2 \phi \sum_{i=1}^K (m + K - 1 - s_i) \alpha_i . \quad (18)$$

Note that the first term gives  $a(\emptyset) = [(m-1)m q_1 \phi]/2$ .

#### Expected Fuel Consumed in the Bolter Queue Recovery Time

Recall that at time  $(m + K - 1)^+$  the number in the bolter queue is random and denoted  $N(m + K - 1)$ . A simple way to calculate the expected fuel consumed in  $B$  is by using absorbing Markov Chain results with  $N(m + K - 1)$  as the random starting state (see Kemeny and Snell [1], Chapter 3).

Suppose we count periods starting with 0 at time  $m + K - 1$ , after which we start emptying any remaining aircraft from the bolter queue. The number of aircraft in the bolter queue at each time after this forms a state of a finite-state Markov Chain with absorbing state 0 and transient states 1, 2, ...,  $m$ . The transient part of the one-step transition matrix is given by

$$Q = \begin{bmatrix} q_2 & & & & \\ p_2 & q_2 & & & \\ & \cdot & \cdot & \cdot & \\ & & \cdot & \cdot & \cdot \\ & & & p_2 & q_2 \end{bmatrix} , \quad (19)$$



which is an  $m \times m$  matrix with  $q_2$ 's on the main diagonal,  $p_2$ 's on the lower diagonal, and zeros elsewhere. If we start in some state, say  $n$ , the total expected fuel consumed will be the total expected number of visits to each state starting in state  $n$ , multiplied by the value of the state. The expected number of visits to each state for each given starting state is given by the fundamental matrix  $(I - Q)^{-1}$ . It is easy to show that for  $Q$  given by (19) we get

$$(I - Q)^{-1} = \frac{1}{p_2} \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & \cdot & & & \\ & & \cdot & \cdot & & \\ & & \cdot & & \cdot & \\ & & & & \cdot & \cdot \\ 1 & & \cdot & \cdot & \cdot & 1 & 1 \end{bmatrix}, \quad (20)$$

an  $m \times m$  matrix with  $1/p_2$  in each position on and below the main diagonal, and zeros above the main diagonal.

Now let  $C$  be the total fuel consumed in emptying the bolter queue with expected value  $c$ . Note that whenever the chain is in state  $i$ , there are  $i$  aircraft in the bolter queue. These consume  $\phi_i$  fuel in a given period. If we write  $N$  for  $N(m + K - 1)$  to simplify the notation, then from (20),

$$E[C|N] = \frac{\phi}{p_2} \sum_{i=0}^{N-1} (N - i) = \frac{(N^2 - N)}{2p_2} \phi.$$

Using equations (9), (13), (14) and (15), after removing the condition on  $N$  we obtain

$$c(S) = \frac{mq_1[2 + (m-1)q_1]\phi}{2p_2} - q_1\phi \sum_{i=1}^K (s_i + 1 - i) - \phi \sum_{i=1}^K [q_1(m-1-s_i+1) - p_2(K-i)] \alpha_i . \quad (21)$$

Note that the first term gives the expected fuel consumption for the no-hole strategy,  $c(\emptyset)$ .

Let  $f$  be the total expected fuel consumed in the recovery of all  $m$  aircraft. If  $f(\emptyset)$  represents this value for the no hole strategy, then from (17), (18) and (21) we get

$$f(\emptyset) = [m(m-1)(p_2 + q_1\phi(q_1 + p_2)) + 2mq_1\phi]/2p_2 ,$$

and

$$f(S) = f(\emptyset) + \sum_{i=1}^K (m+i-s_i-1)(1 - \phi(q_1 + p_2)\alpha_i) . \quad (22)$$

The expressions (22) are used to calculate the total expected fuel consumed once the  $\alpha_i$ 's,  $i = 1, 2, \dots, K$  have been calculated for a given strategy. Note that if  $p_1 = p_2$  and  $\phi = 1.0$ , then all terms in the sum in (22) are non-negative. Thus if the fuel consumption in the bolter queue were the same as that in the holding pattern, and if the probability of landing were the same for each queue, then the optimal strategy would

be to leave no holes when trying to minimize total expected fuel consumed.

In Section 7 equations (10), (16) and (22) are used in numerical computations to investigate various landing strategies. All these require computation of the  $\alpha_i$ 's, and these are investigated in the next section.

## 6. PROBABILITY CALCULATIONS

Before numerical calculations can be carried out with the expressions derived so far, we need to be able to evaluate the  $\alpha_i$ 's,  $i = 1, 2, \dots, K$ . Of course for the no-hole strategy there are none to calculate, so we assume that  $K \geq 1$ . In this section we present an algorithm to evaluate the probabilities recursively.

Consider a strategy  $S = \{s_1, s_2, \dots, s_K\}$  for a given  $K \geq 1$ , where  $s_k$  is the  $k$ th hole. Now define

$$\begin{aligned} d_1 &= s_1 - 1, \\ d_k &= s_k - s_{k-1} - 1, \quad 2 \leq k \leq K, \\ D_k &= \sum_{i=1}^k d_i = s_k - k, \quad 1 \leq k \leq K. \end{aligned}$$

Also define the binomial probability mass function

$$\begin{aligned} b(j; n, p) &= \binom{n}{j} p^j (1-p)^{n-j}, \quad 0 \leq j \leq n \\ &= 0 \quad n < j. \end{aligned}$$

Finally define

$$g_k(j) = P[N(s_k - 1) = j], \quad 0 \leq j \leq D_k$$

and

$$h_k(j) = P[N(s_k) = j], \quad 0 \leq j \leq D_k.$$

Thus  $g_k(j)$  gives the probability of having  $j$  aircraft in the bolter queue immediately preceding the  $k$ -th hole, and  $h_k(j)$  gives a similar probability immediately after the  $k$ -th hole. Clearly  $\alpha_k = 1 - g_k(0)$ ,  $1 \leq k \leq K$ .

The following algorithm can be used to calculate the  $g_k$ 's recursively:

#### Algorithm

1. Set  $g_1(j) = b(j; d_1, q_1)$ ,  $0 \leq j \leq D_1$   
and  $k = 1$ .

2. Set

$$h_k(j) = \begin{cases} g_k(j) + g_k(j+1)p_2, & j = 0, \\ g_k(j)q_2 + g_k(j+1)p_2, & 1 \leq j \leq D_{k-1}, \\ g_k(j)q_2, & j = D_k, \\ 0 & D_k < j. \end{cases}$$

3. Set

$$g_{k+1}(j) = \sum_{i=1}^j h_k(j-i) b(i; d_{k+1}, q_1), \quad 0 \leq j \leq D_{k+1}$$

4. If  $k+1 < K$ , set  $k$  equal to  $k+1$  and return to Step 2.  
Otherwise go to Step 5.

5. Set  $\alpha_k = 1 - g_k(0)$ ,  $1 \leq k \leq K$ .

6. Stop.

The calculations in this algorithm follow directly from the definitions of the probabilities.

## 7. NUMERICAL EXAMPLES

In this section we use equations (10), (16) and (22) to demonstrate the effects of different landing strategies on the expected recovery time  $r$ , the variance of this time  $v$ , and the expected fuel consumed in recovery  $f$ , respectively. The following (typical) parameter values are used in all calculations:

Number of aircraft,  $m = 30$ ,

Bolter queue fuel ratio,  $\phi = 1.4$ .

We also restrict ourselves to strategies with "equally spaced holes." For example, if we place a hole at every eighth position, we would have  $S = \{7, 15, 23, 31\}$ , and from the definitions in Section 6,  $d_i = 7$ ,  $i = 1, 2, \dots, K$  with  $K = 4$ . These strategies are ones which are usually used in practice, and the appropriate choice of  $d_i$ , as  $p_1$  and  $p_2$  vary, is discussed. The value of  $d_i$  gives the number of aircraft between holes  $i-1$  and  $i$ . Note that the number of aircraft after the last hole cannot exceed  $d_i$ , and is equal to it only when  $m$  is divisible by  $d_i$ .

In the first set of calculations we assume that the probability of landing successfully from the bolter queue is the same as that from the holding pattern; i.e.  $p_2 = p_1$ . Table 1 shows the values of  $r$ ,  $v$ , and  $f$  for  $d_1 = 1, 2, \dots, 10, 15$ , and the "no-hole" case ( $d_1 = 30$ ), for values of  $p_1 = p_2 = 0.5, 0.6, 0.7, 0.8, 0.9$ . The results are plotted in Figures 3, 4 and 5.

The results for  $r$  illustrate our previously observed result, that it is minimized by using the no-hole strategy. However, one can see from the curves in Figure 3 how insensitive the expected recovery time is to the strategy used. For example,  $p_1 = p_2 = 0.7$  there is virtually no change in expected recovery time for all strategies with  $d_i \geq 4$ . Notice that in all cases shown there is a sharp increase in  $r$  for very small values of  $d_i$ .

The results for  $v$  are shown in Figure 4. Again we see how insensitive  $v$  is to  $d_i$  for  $d_i$  large, but how sensitive it is for  $d_i$  very small. These curves illustrate the point made at the end of Section 4, that  $v$  can be reduced at the expense of increasing  $r$ . For example, if  $p_1 = p_2 = 0.6$ , the no-hole strategy gives an expected recovery time of 49.0 units with a variance of 33.3. With  $d_i = 5$  (a hole every sixth position),  $r$  increases to 49.1 and the variance is reduced to 32.6; both are insignificant changes. But if  $d_i = 1$  (a hole every second position), then  $r$  increases to 59.9 while  $v$  is reduced to only 5.4.

The results for  $f$  are shown in Figure 5. Note that for  $p_1 \leq 0.8$  there is a minimum point on each curve (the minimum for  $p_1 = 0.9$  lies outside the plotted range of  $d_i$ ). These minima have been marked for clarity. For example, if  $p_1 = p_2 = 0.7$  the strategy which minimizes the expected fuel consumed is to leave a hole in every fifth position ( $d_i = 4$ ), or  $S = \{4, 9, 14, 19, 24, 29, 34\}$ . Note that with this



strategy there is very little increase in the expected recovery time, and a small decrease in the variance, when compared to the no-hole strategy. Note however that the expected savings in fuel is only 23 units (714-691), or 3.2% of that used with the no-hole strategy. Thus the optimum "equally spaced hole" strategy gives little improvement over the simple no-hole strategy for  $p_1 = p_2 = 0.7$ ,  $m = 30$ , and  $\phi = 1.4$ . The reader can see from Figure 5 that this result holds quite generally.

The small fuel savings seen in the calculations given in Table 1 make it questionable as to whether or not it is worth searching for an optimal strategy among those that do not have equally-spaced holes. Although this could certainly be done using (22) we do not pursue this route, but consider another alternative not yet discussed in the paper. A number of aircraft are now fitted with automatic landing systems, which when operated, make landing virtually certain. There are a number of reasons why these devices are not always used. Constant use would lead to a reduction in landing skills by the pilot. This would lead to a dangerous condition in case of failure of the automatic system, and would also lead to pilot discontentment and motivation problems. A reasonable question to ask is what strategy should be used with regard to the automatic landing system.

Equations (10), (16) and (22) can be used to demonstrate the effect of using the automatic landing system when making an attempt from the bolter queue. To do this we repeat our previous calculations with the condition  $p_2 = p_1$  replaced by



$P_1$ $d_1$	0.5			0.6			0.7			0.8			0.9		
	$\bar{r}$	v	f	r	v	f	r	v	f	r	v	f	r	v	f
1	64.1	28.3	1045	59.9	5.4	939	58.8	1.3	900	58.3	0.4	884	58.1	0.1	875
2	59.6	55.5	1023	50.6	24.3	830	45.7	6.8	722	43.9	1.4	673	43.3	0.3	654
3	59.2	58.5	1039	49.5	30.3	630	43.1	12.8	695	39.7	3.2	617	38.4	0.5	583
4	59.1	59.3	1050	49.2	31.9	836	42.4	15.4	691	38.2	4.7	598	36.4	0.6	551
5	59.0	59.6	1057	49.1	32.6	842	42.2	16.7	693	37.4	6.4	591	34.9	1.2	533
6	59.0	59.8	1063	49.0	32.9	847	42.0	17.3	695	37.1	7.3	589	34.1	1.3	523
7	59.0	59.9	1066	49.0	33.0	850	42.0	17.6	697	36.9	7.6	588	33.9	1.4	517
8	59.0	59.9	1069	49.0	33.1	853	42.0	17.9	699	36.8	8.1	588	33.4	2.0	513
9	59.0	60.0	1072	49.0	33.2	855	41.9	18.0	701	36.7	8.4	588	33.2	2.0	510
10	59.0	60.0	1074	49.0	33.3	857	41.9	18.1	703	36.6	8.7	589	32.9	2.6	509
15	59.0	60.0	1080	49.0	33.3	863	41.9	18.3	708	36.5	9.2	593	32.5	3.2	506
30 (no hole)	59.0	60.0	1086	49.0	33.3	869	41.9	18.4	714	36.5	9.4	598	32.3	3.7	507

Table 1. Calculations for the case  $P_2 = P_1$ ,  $m = 30$ ,  $\phi = 1.4$ .

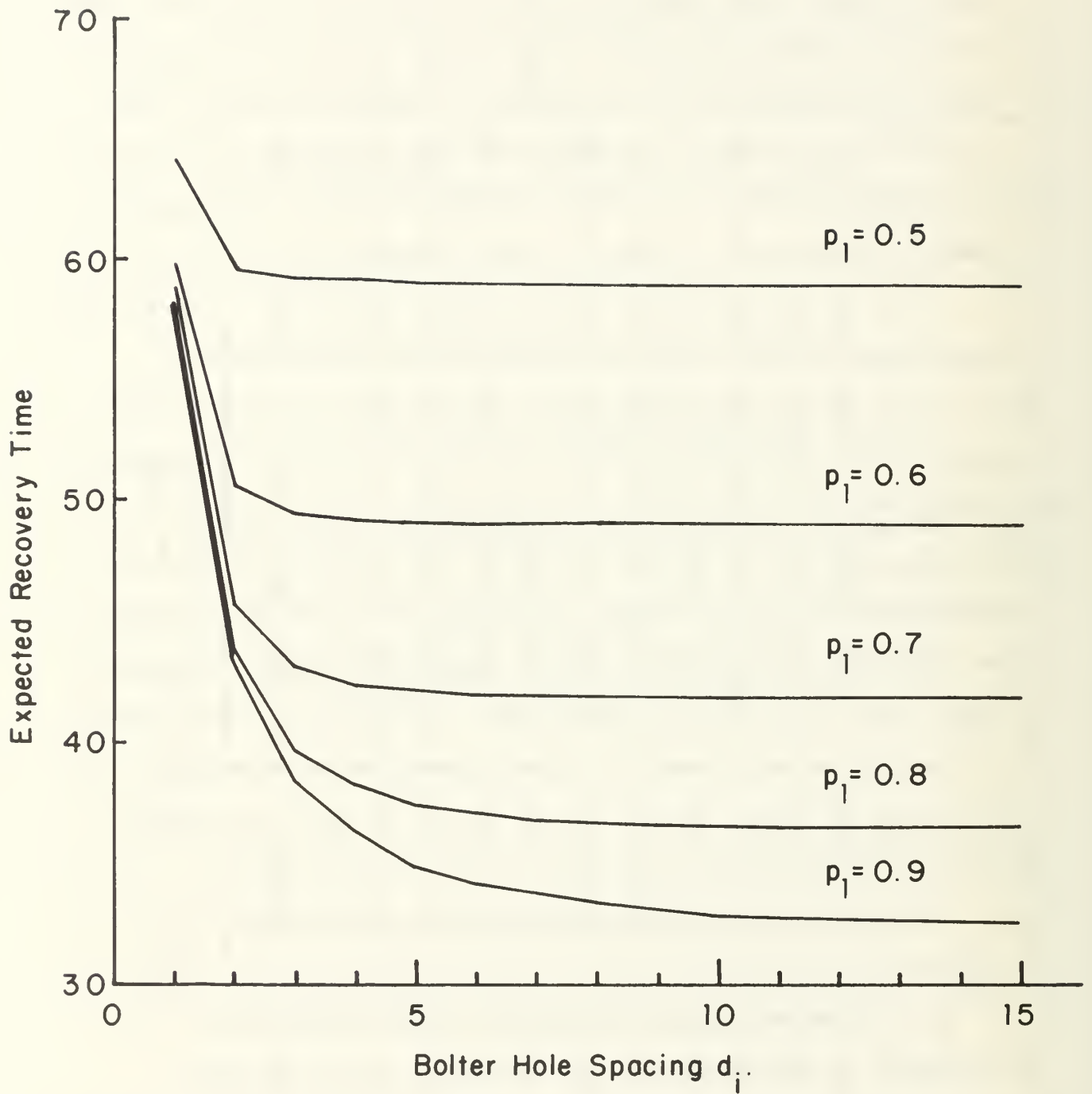


Figure 3: Expected Recovery Time with  $p_2 = p_1$ ,  $m = 30$ .

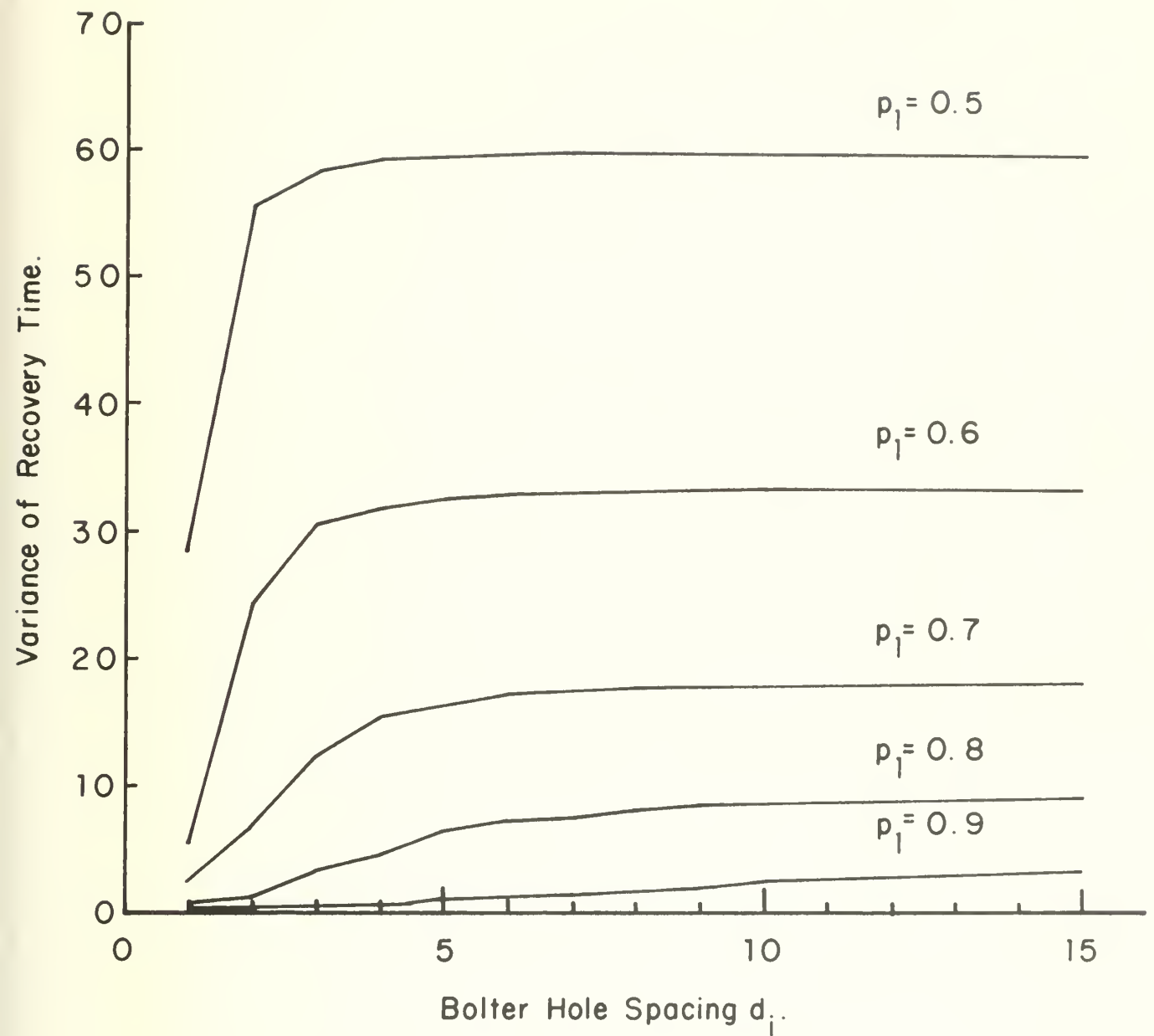


Figure 4: Variance of Recovery Times with  $p_2 = p_1$ ,  $m = 30$ .

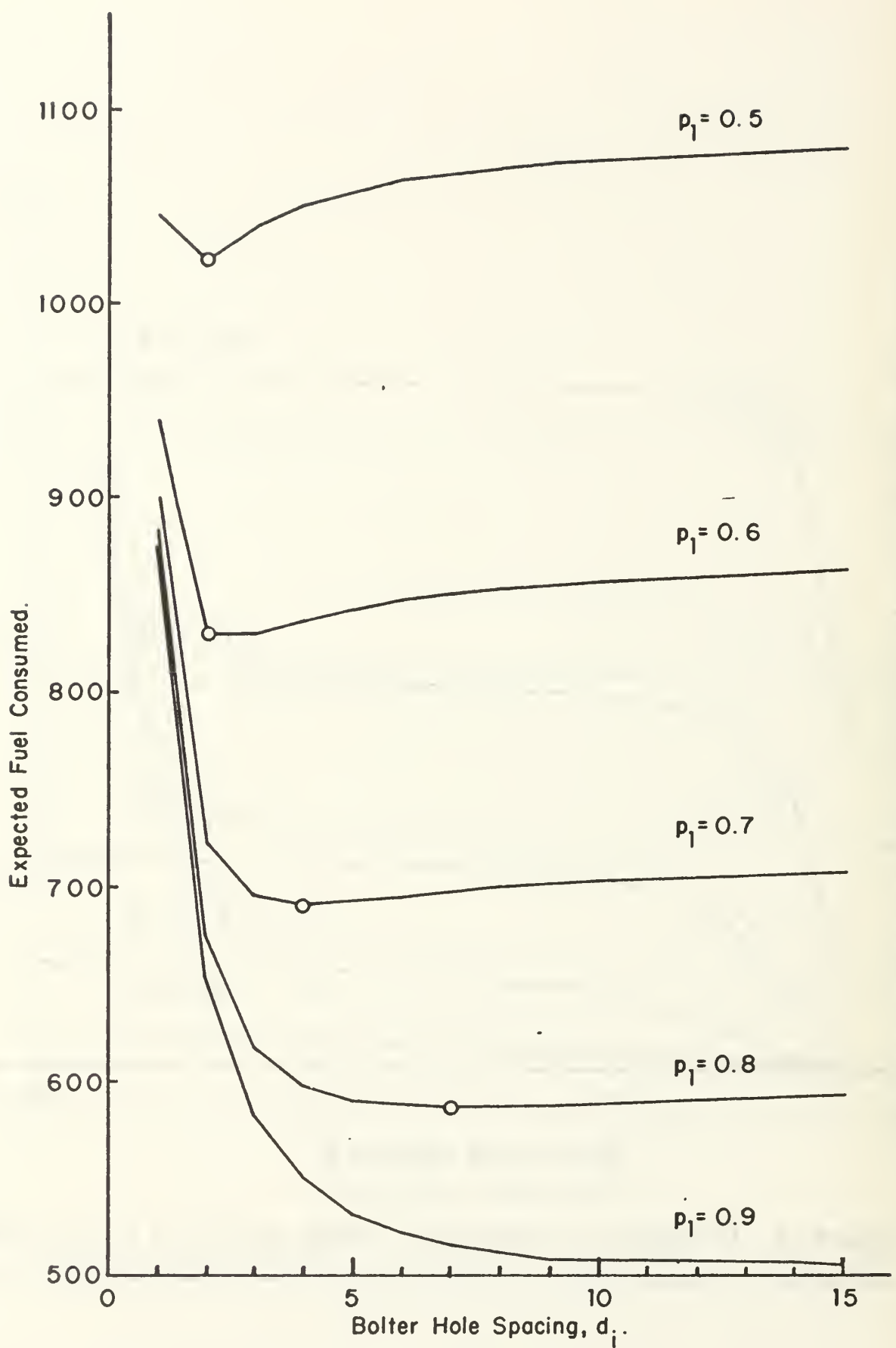


Figure 5 : Expected Fuel Consumed with  $p_2 = p_1$ ,  
 $m = 30$ ,  $\phi = 1.4$

$P_1 \backslash d_1$	0.5			0.6			0.7			0.8			0.9		
	r	v	f	r	v	f	r	v	f	r	v	f	r	v	f
1	58.5	0.3	891	58.4	0.2	887	58.3	0.2	883	58.2	0.2	878	58.1	0.1	874
2	45.7	3.1	744	44.5	1.4	700	43.8	0.7	676	43.5	0.4	661	43.2	0.2	652
3	44.3	6.4	778	41.8	4.6	693	39.9	2.3	634	38.9	0.9	599	38.3	0.3	581
4	44.1	7.1	810	41.3	6.1	710	38.8	3.9	631	37.1	1.4	577	36.3	0.3	547
5	44.0	7.3	832	41.1	6.7	726	38.4	5.0	637	36.1	2.5	570	34.7	0.7	529
6	44.0	7.4	848	41.1	6.9	738	38.2	5.5	643	35.7	3.1	568	33.9	0.9	519
7	44.0	7.5	858	41.0	7.0	746	38.1	5.7	649	35.5	3.4	568	33.6	0.8	512
8	44.0	7.5	867	41.0	7.1	753	38.1	5.9	653	35.3	3.8	570	33.1	1.2	508
9	44.0	7.5	873	41.0	7.1	759	38.0	6.0	657	35.2	4.0	571	32.9	1.2	506
10	44.0	7.5	880	41.0	7.2	764	38.0	6.1	662	35.1	4.3	573	32.6	1.7	505
15	44.0	7.5	896	41.0	7.2	778	38.0	6.3	673	35.0	4.6	580	32.2	2.2	503
30	44.0	7.5	913	41.0	7.2	793	38.0	6.3	685	35.0	4.8	590	32.0	2.7	506
(no hole)															

Table 2. Calculations for the case  $p_2 = 1.0$ ,  $m = 30$ ,  $\gamma = 1.4$ .

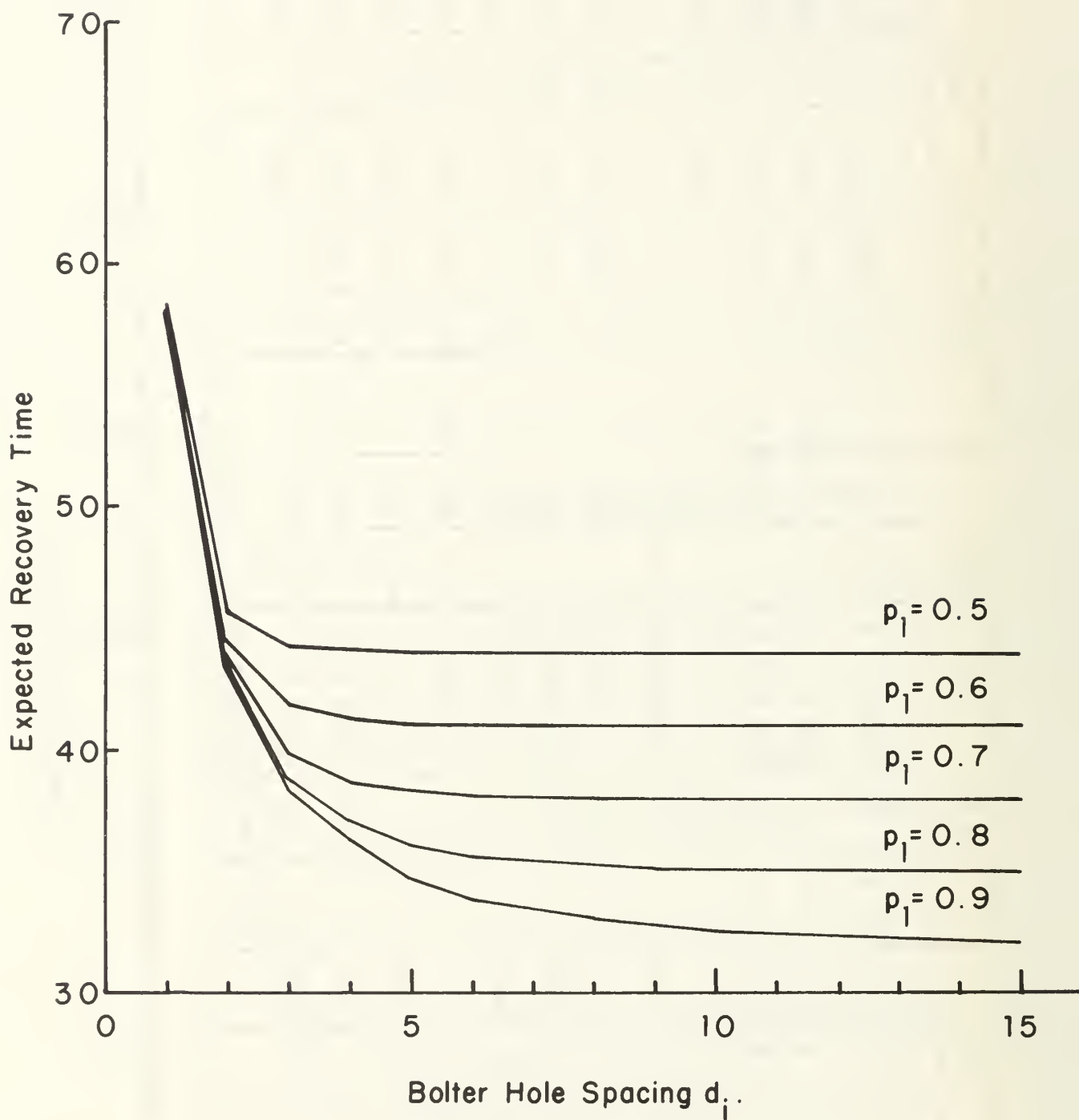


Figure 6: Expected Recovery Time with  $p_2 = 1.0$ ,  $m = 30$ .

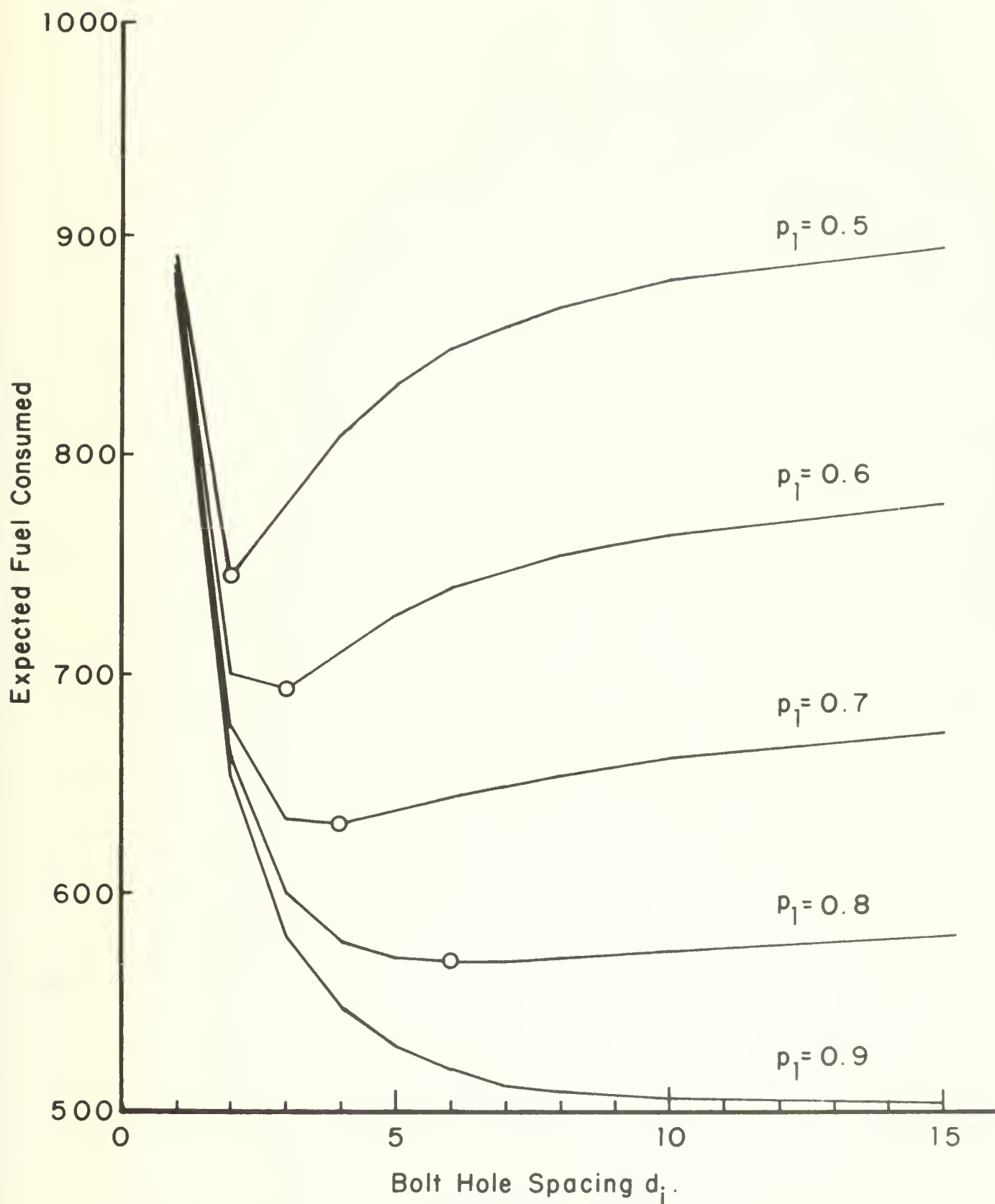


Figure 7: Expected Fuel Consumed,  $p_2 = 1.0$ ,  $m = 30$ ,  $\phi = 1.4$

$p_2 = 1.0$ . The results are given in Table 2, and are plotted in Figures 6 and 7.

Figure 6 shows the expected recovery times for various values of  $p_1$ , as a function of the bolter hole spacing. The same general shape is observed as was found in Figure 3 for  $p_2 = p_1$ . However, for  $p_1$  small (0.5 - 0.6) there is a considerable reduction in recovery time as one might expect. These curves can be used to measure this reduction. For example, if  $p_1 = 0.6$  (possibly from bad weather conditions at night) and a strategy is used with  $d_i = 5$ , the expected recovery time can be reduced from 49.1 time units to 41.1, a reduction of 10 minutes (16.3%) by using the automatic landing device on attempts from the bolter queue.

A look at Table 2 will show the reader that considerable reductions in the variance of recovery time are achieved with  $p_2 = 1.0$ . Because the variances are so small in all cases, they have not been plotted.

Figure 7 shows the expected fuel consumed in recovery for various values of  $p_1$ , and these curves should be compared with those in Figure 5. Notice that they have the same basic shape and that the minimum points have been marked. The minima are more pronounced in Figure 7, showing a greater percentage savings in fuel over the no-hole strategy. For the case described above with  $d_i = 4$ ,  $p_1 = 0.7$ ,  $m = 30$ ,  $\phi = 1.4$ , but with  $p_2 = 1.0$ , the expected savings in fuel is 54 units (685-631), or 7.9% compared with only 3.2% for  $p_2 = p_1$ .



Not all possible strategies have been analyzed in this paper. We have presented enough to illustrate the output which can be obtained from the model. It should be quite straightforward to program on a hand-held calculator the relevant equations, so that instant calculations can be made aboard a carrier with very little resources. In this way any conditions prevalent on a given recovery can be taken into account.

## 8. ACKNOWLEDGMENT

The bolter hole spacing problem was brought to the author's attention by LT Roger K. Hull, USN, who is himself a pilot, and who recognized the necessity for studying this problem. The author greatly appreciates the help given by LT Hull in explaining the problem. The work was completed while the author was on sabbatical leave at the London School of Economics.

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